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INTERFEROMETER STUDY

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RADIATIONS IN A MAGNETIC FIELD

BY

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A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF WISCONSIN

1899

[Reprinted from the PHYSICAL REVIEW, Vol. IX., Nos. 1 and 2, 1899.]

PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA.

PREFATORY NOTE.

The historic interest in the class of experiments to which the following research belongs, consists in the importance of establishing the electromagnetic theory of light upon an experimental basis.

The first link in this connecting chain is furnished by Faraday's discovery of the magnetic rotation of the plane of polarization. A second link is furnished by the experimentally determined ratio of electromagnetic to electrostatic units as expressed by the equation

$$V = 1/\sqrt{Km}$$

where V = velocity of light in a non-conducting medium;

K = dielectric inductive constant of the medium;

m = magnetic inductive constant of the medium.

A third link is furnished in Hertz' epoch-making experiments on electrical waves. The influence of a magnetic field of force upon a source of light placed within it, is a fourth link, and marks one of the most important advances that has been made since the electromagnetic theory was first set forth.

A further importance is to be attached to such experiments in that much light will be thrown upon the motions of the ultimate particles of matter, and possibly upon the ultimate structure of matter itself.

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AN INTERFEROMETER STUDY OF RADIATIONS IN A MAGNETIC FIELD. I.

JOHN C. SHEDD.

PART ONE. HISTORICAL SURVEY.

IN 1845 Faraday discovered the rotation of the plane of polarization due to a magnetic field of force. This, perhaps, suggested the further experiment as to the effect of a magnetic field upon a source of radiation placed within it.¹

In the light of what is now known Faraday's repeated failure to obtain positive results from this last experiment may be ascribed to the low dispersive power possessed by his apparatus, and to the comparatively low temperature of the gas flame used as a source of radiation.

In 1865 Maxwell propounded the electromagnetic theory of light, which not only correlated all hitherto observed phenomena, but also furnished a scientific basis for future research.

In 1875 Professor Tait² presented a paper "On a Possible Influence of Magnetism on the Absorption of Light" which, while not realized by him experimentally is of interest in this connection. He says in part :

"The explanation of Faraday's rotation of the plane of polarization of light by a transparent diamagnetic requires, as shown by

¹ Appendix I.

² Proc. Roy. Soc. Edinburgh. Sessions 1875-76, p. 168.

Thomson,¹ molecular rotation of the luminiferous medium. The plane polarized ray is broken up, while in the medium, into its circularly polarized components, one of which rotates with the ether so as to have its period accelerated, the other against it in a retarded period. Now, suppose the medium to absorb one definite wave-length only, then—if the absorption is not interfered with by the magnetic action—the portion absorbed in one ray will be of a shorter, in the other, of a longer period than if there had been no magnetic force; and thus, what was originally a single dark absorption line might become a double line, the components being less dark than the single one."

The line of reasoning here presented, if applied to a source of radiation (instead of absorption) placed in a magnetic field would give an analogous solution of two bright lines of less intensity than the original line.

The first experimental results in this field of work were obtained by M. Fievez² in 1885–86. His observations consisted in a broadening of the spectral line, and increased brilliancy of illumination. No observations are recorded as to the state of polarization.

The phenomena of reversal and increased brightness of the lines, while undoubtedly precipitated by the action of the magnetic field, were probably due, primarily, to changes of temperature and density. The broadening of the line, however, was undoubtedly a genuine magnetic effect, and as Preston³ remarks, had Fievez been acquainted with the theory of the subject the whole question would have been settled in 1885.

Fievez does not seem to have been familiar with what Faraday had done as regards the state of polarization, nor to have taken any special precautions against spontaneous reversals: yet, while his work may, in the light of recent investigations, appear meager, it deserves an important place in the history of the subject, and must sooner or later have led to the very results reached by Zeeman.

A paper entitled "Causes of Double Lines and Close Satellites

¹ Reprint Thomson's Papers on Electrostatics and Magnetism. Second Edition, p. 423, Footnote.

² Appendix II.

³ Phil. Mag. (5), 45, 1898, p. 338.

in the Spectra of Gases," by G. J. Stoney,¹ though not considering the special case of magnetic forces, has, nevertheless, a theoretical bearing upon the subject..

Of still greater importance are the writings of Lorentz,² published in 1892 and 1895, inasmuch as they were a guide to Zeeman in his experiments.

In March, 1897, Dr. P. Zeeman³ communicated to the Philosophical Magazine the results of a research that has proved wonderfully fruitful in his own hands and also in the hands of others. Zeeman was familiar with Faraday's experiments, but did not know of Fievez's work. He was influenced, as a matter of course, by Maxwell's electromagnetic theory of light, and in particular was guided by Lorentz's exposition of it. His first experiments were identical with those of Faraday, excepting that he had a Rowland grating of 14,000 lines to the inch, and hence had a much greater dispersion. With this arrangement of the apparatus, a broadening of the spectral line was observed, similar to that seen by Fievez. Guided, however, by Lorentz's theory, Zeeman tested the state of polarization of the broadened line, and found that, when viewed parallel to the lines of force, the edges of the line were circularly polarized as predicted by Lorentz's theory.

Lorentz's Theory.—In this theory it is assumed that light vibrations are the vibrations of electrically charged ions of definite mass. Thus suppose such an ion having a charge e and mass m , to vibrate with simple harmonic motion about its center of equilibrium. Such an ion moving in a magnetic field would then experience mechanical forces, which would cause a change of period of vibration. The amount of this change of period would depend upon the ratio e/m , and the measurement of the change of period would give a knowledge of this ratio.

The equations giving the modified period are derived as follows.⁴

In Fig. 1 let the origin be at o and the axis of Z be parallel to

¹ Sci. Trans. Roy. Dublin Soc., Vol. IV., p. 563, 1891.

² Lorentz, "La Theorie electromagnetique de Maxwell," Leyden, 1892. "Versuch einer Theorie der electrichen und optischen Erscheinungen in bewegten Korpern," Leyden, 1895.

³ Phil. Mag. (5), 43, p. 226. Same art. Astro. Phys. Jr., 5, p. 332, 1897.

⁴ See Zeeman, Phil. Mag. (5), 43, p. 226.

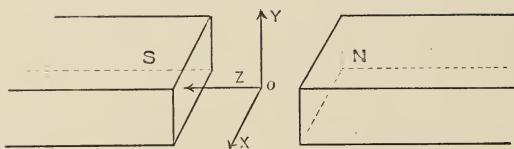


Fig. 1.

the lines of force, the axes of X and Y being perpendicular to this direction. Then if H be the strength of field, the equations of motion relative to the X and Y axes are

$$\left. \begin{aligned} m \frac{d^2x}{dt^2} &= -K^2x + eH \frac{dy}{dt} \\ m \frac{d^2y}{dt^2} &= -K^2y - eH \frac{dx}{dt} \end{aligned} \right\} \quad (1)$$

The term K is the coefficient of elasticity of the ion, the second term gives the mechanical force due to the magnetic field.

The solution of these equations yields for the period of vibration the following values :

$$\text{If } H = 0, \quad T = \frac{2\pi\sqrt{m}}{K} \quad (2)$$

If H be not zero, and we regard all forces in the XY plane as symmetrical with respect to the axis of Z , then

$$T' = \frac{2\pi\sqrt{m}}{K} \left(1 \pm \frac{eH}{2K\sqrt{m}} \right). \quad (3)$$

The meaning of equations (2) and (3) is as follows :

If we analyze along the three coördinate axes, the motion of the moving ion, which by virtue of this vibration is emitting light, we shall have three component vibrations of equal period. We may next compound the two components which lie in the XY plane into circular motion, and since we do not know the sense of rotation, we may conceive it to be in both directions simultaneously.¹

If the source of light be viewed in a direction parallel to the magnetic field, the component whose motion is parallel to the axes of Z is not effective, and the ray is, therefore, composed of circularly

¹ The sense of rotation will depend upon the sign of the charge e upon the iron.

polarized components, whose periods of rotation are the same, and for zero magnetic field have the value given by equation (2). If the magnetic field be equal to H , then the periods become changed, and are given by equation (3). The periods being changed by equal and opposite amounts, the original spectral line becomes two lines of equal intensity, symmetrically situated with respect to the original unmagnetized line; the components being circularly polarized in opposite senses.

Viewed in a direction normal to the magnetic field, the component parallel to the axis of Z , and, therefore, parallel to the axis of the magnetic whirl, is not affected by it, is unaltered in period and appears as a plane polarized ray. The two circular components, being now viewed parallel to the $X Y$ plane, also appear as plane polarized rays, the plane of polarization being normal to that of the Z component.

Thus viewed, the spectral line is broken up into *three* component lines, and if the magnetic field is of sufficient intensity there will be seen *three* distinct lines, the two outlying ones having their vibrations in a plane normal to the magnetic field, and the central one in a plane parallel to it. If the magnetic field is not powerful enough to separate the three components they will overlap, and the line appears merely broadened. The components may, however, be isolated by means of a nicol prism.

In his earlier work¹ Zeeman obtained only a broadened line, but by means of the nicol he was able to test the state of polarization; and, later,² he succeeded in obtaining both the doublet parallel to the magnetic field, and the triplet normal to the field, as predicted by Lorentz's theory.

Methods.—In the further progress of the work two radically different methods have been developed.

The first is that followed by Zeeman and many other investigators. It consists in photographing the spectral lines, and in measuring the separation of the "magnetized" components by means of the micrometer dividing engine. The chief merit of this method is that a permanent record is secured in the photograph. It is also

¹ Phil. Mag. (5), 43, p. 226.

² Phil. Mag. (5), 44, pp. 55 and 255; 45, p. 197.

an advantage that the phenomenon is observed directly. The limitations of the method are : *First*, the fact that the quantity to be measured is minute, and the micrometer method of direct measurement is of necessity limited in range ; *Second*, it frequently happens that the components, whose distances apart are to be measured, are so nebulous as to make it exceedingly difficult to make exact micrometer settings. *Third*, the time of exposure necessary is sometimes so long as to make the method prohibitory.

The second method is due to Professor A. A. Michelson, and may be called the *Interferometer Method*. It has been used successfully with magnetic fields far too weak to give any sensible effect by the direct method, and has been shown to have a delicacy and sensitiveness far in excess of any photograph. The method, as used by Professor Michelson, consists in obtaining the visibility curves of the spectral line, both when unmagnetized, and also with fields of different intensity. These visibility curves are then analyzed,¹ and the distribution of light at the source of illumination obtained. This gives directly the various "magnetic" components of the line : By means of a nicol prism the two planes of polarization may be separately examined.

The advantages of this method are—briefly—*First*, the visibility curve enables the separation of lines not hitherto resolved by any other method. *Second*, the eye is the instrument of investigation, and hence there is no need of long exposure as in the case of taking photographs. *Third*, any change of polarization—or other effect—taking place during the period of observation may be detected, while the photographic process is necessarily an integrating method.

The disadvantages are : *First*, the method is an indirect one, *i. e.*, the observations are made not on the lines themselves but on interference fringes produced by them. *Second*, the accurate estimate of a visibility curve is by no means an easy matter, and the rare success attained by Professor Michelson has been equalled by no one else, and can only be approached by practice and great patience. *Third*, the record of the instrument is not automatic, as in the photograph, and is subject to the personal error of the observer. *Fourth*, the reflection from the half silvered surface of the in-

¹ Phil. Mag. (5), 34, p. 280, 1892. Astro. Phys. Jr., 7, p. 129, 1898.

terferometer affects the two beams polarized in perpendicular planes, to a different degree, so that when both beams are simultaneously observed they have not their normal ratio of brightness, with the result that the fringes are correspondingly deceptive.

In the hands of an experienced observer the interferometer is undoubtedly the most powerful instrument of attack that is available at present, unless, indeed, Professor Michelson has presented in the Eschelon Plate Spectroscope, an instrument of equally great value. Under circumstances less favored than those enjoyed by Professor Michelson, it is difficult to see how the Interferometer Method as he uses it, can be successfully used. There are, however, modifications rendering this instrument more available, which have been used by the present writer.

Experimental Results.—The agreement between theory and experiment presented by Zeeman's early experiments was truly remarkable, and the apparent simplicity of the phenomenon seemed equally worthy of notice. This apparent simplicity, however, was soon found not to be true of all spectral lines, and more complex forms were found. Exceptions were also found to the state of polarization as first described by Zeeman.

The first observer to note a departure from the normal form was M. Cornu.¹ His apparatus was similar to that of Zeeman excepting that he used a double image prism, and was thus able to observe both planes of polarization simultaneously. For a magnetic field strength of 13,000 C. G. S. units he observed that the sodium line, D_1 when viewed normal to the magnetic field was a quadruple, the inner components being polarized perpendicular to the magnetic field and the outer ones parallel to this direction. The line, D_2 he found to be a hazy triple with each member perhaps doubled.

M. Cornu was soon followed by others. Preston² succeeded in photographing as many as five different types, and Michelson³ with the interferometer showed three well marked groups of lines.

As regards polarization Becquerel and Deslandres⁴ have found

¹ Astro. Phys. Jr., 6, p. 378, 1897; 7, p. 163, 1898.

² Phil. Mag. (5), 45, p. 330, 1898; 47, p. 165, 1899.

³ Phil. Mag. (5), 44, p. 109, 1897, same Art. Astro. Phys. Jr., 6, p. 48, 1897; Phil. Mag. (5), 45, p. 348, 1898, same Art. Astro. Phys. Jr., 7, p. 131, 1898.

⁴ C. R., April 4, 1898.

that one of the iron lines when viewed perpendicularly to the magnetic field becomes a triple, in which the usual state of polarization is reversed. The same phenomenon has also been observed at the Johns Hopkins¹ University.

The classification of lines as given by Preston is shown in Fig. 2, who thus describes them.

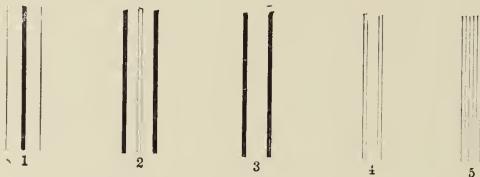


Fig. 2.

"In 1 we have the normal triplet. In 2 we have the weak middled 'quartet' in which nearly all the light is concentrated in the two side lines. Next we have in 3 the doublet in which the central line has completely disappeared. Next in 4 we have the double doublet, or two pairs of fine lines, and finally in 5 the sextet or three pairs of equally spaced sharp lines."

Preston² cites the following as examples of the above types

Type 1, Cd, 4678; Mg, 5167; Zn, 4680 and the vast majority of other lines. Type 2, Mg, 5183; Cd, 5086; Zn, 4810. Type 4, Mg, 5173; Cd, 4800; Zn, 4722. In his latest work³ Preston seems to restrict his classification to three types, viz.: "Diffuse triplets," "quartets" and "pure triplets." It may be that types 3 and 4 are modified forms of the same type, as also 2 and 5. This would leave but three types.

The types found by Michelson⁴ are shown in Fig. 3. The upper curves are taken by the interferometer and the lower by the Echelon plate spectroscope. The spectral lines are viewed normal to the magnetic field in both figures 2 and 3.

¹ Astro. Phys. Jr., 8, p. 48, 1898.

² Phil. Mag. (5), 45, p. 330.

³ Phil. Mag. (5), 47, p. 178. See also Nat., Vol. 59, p. 226, where seven types are given.

⁴ Astro. Phys. Jr., 7, p. 136, 1898. Nat., March 9, 1899.

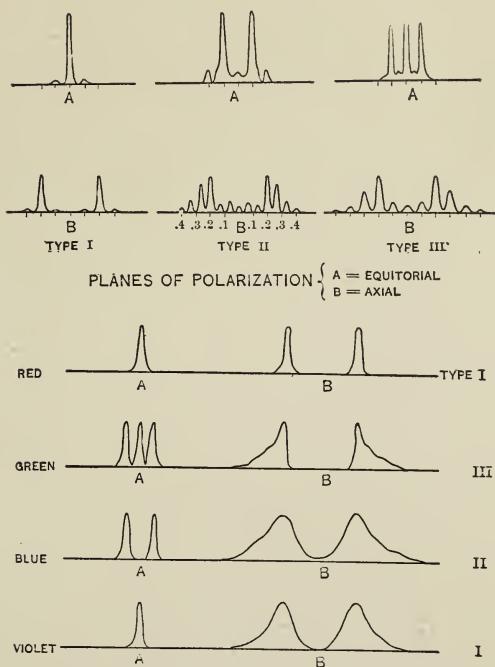


Fig. 3.

The following are some examples :

Type I.	Type II.	Type III.
Hg yellow line.	Hg violet line.	Hg green line.
Cd red line.	Cd blue line.	Cd green line.
Zn red line.	Zn blue line.	Mg green line (5183).
Au green line.	Na yellow line.	
Ag yellow line.	Au yellow line.	
Ag green line.		

Michelson adds a fourth type in which a broad or complex line is simplified or narrowed in the magnetic field. Examples of this are Cu yellow line and Mn green line (5340). This effect is true of the central member of the triplet in the case of these two lines.

If now a comparison be made between such lines as have been observed by several persons the following rather meager data are found.

TABLE I.

Lines.	Type.	Field Strength.	Observer.
Zn. 4810.	Diffuse triplets.	—	Preston.
" "	Type II	10,000.	Michelson.
Cd. 5086.	Diffuse triplets.	—	Preston
" "	Type III.	10,000.	Michelson.
" 4800.	Quartets.	—	Preston.
" "	Type II.	10,000.	Michelson.
Na. D ₁	Type II.	10,000.	Michelson.
" "	Quartet.	13,000.	Cornu.
" D ₂	Type II.	10,000.	Michelson.
" "	Sextet.	13,000.	Cornu.

The results of such a comparison are of little value, unless the conditions as regards the source of radiation and the magnetic field strength are known to be the same. Two things are, however, apparent.

I. The phenomenon is by no means as simple as was at first supposed by Zeeman.

II. The superiority of the Interferometer Method as regards resolving power is shown.

Modified Theories.—Having found that the extremely simple deductions from Lorentz's theory, as made by Zeeman, do not comprehend the observed phenomena, it becomes necessary, either to present a new hypothesis or suitably to modify the old one. This modification has been made by Lorentz,¹ Larmor² and others, while Preston³ has pointed out that the paper of Dr. Stoney,⁴ already cited, anticipates the desired theory.

Dr. Stoney considers the effect of perturbing forces upon an ion, moving in an elliptical orbit under the action of a central force, which is proportional to the distance. If the perturbing force be such as to cause the orbit to rotate in its own plane, then the spectral line becomes a doublet. Thus if the ion be moving with an

¹ Proc. Roy. Acad. Sci. Amsterdam, June 25, 1898. Also Astro. Phys. Jr., 9, p. 37. Wied. Ann., Bd. LXIII., p. 278, 1897.

² Phil. Mag. (5), 44, p. 503.

³ Phil. Mag. (5), 47, p. 171.

⁴ Trans. Roy. Soc. Dublin, Vol. IV., p. 563. See also Preston, Phil. Mag. (5), 47, p. 171.

angular velocity Ω then $\Omega = 2\pi N$ where N is the frequency of the rotation. If now an angular velocity of ω be impressed upon the system as a whole, and in the same plane as Ω , then the resultant angular velocity is the algebraic sum of the two. The resultant frequency will also be the algebraic sum; and since Ω is to be regarded in both senses, the resultant motion will have a double frequency of $N + n$ and $N - n$.

Viewed dynamically the equations show what forces are necessary to produce the above changes. If the ion rotate with an angular velocity Ω , and the orbit itself rotate about an axis passing through the center of force, and having a direction (lmn) with an angular velocity ω , then the component velocities referred to the rotating orbit are

$$\left. \begin{aligned} u &= \frac{dx}{dt} = -\omega ny + \omega mz \\ v &= \frac{dy}{dt} = -\omega lz + \omega nx \\ w &= \frac{dz}{dt} = -\omega mx + \omega ly \end{aligned} \right\} \quad (4)$$

The component accelerations are

$$\left. \begin{aligned} \frac{du}{dt} &= -\omega nv + \omega mw \\ \frac{dv}{dt} &= -\omega lw + \omega nu \\ \frac{dw}{dt} &= -\omega mu + \omega lv \end{aligned} \right\}. \quad (5)$$

Expanding equation (5) from equation (4) the acceleration along the axis of X is

$$\begin{aligned} \frac{du}{dt} &= \frac{d^2x}{dt^2} = -\omega n \frac{dy}{dt} + \omega m \frac{dz}{dt} - \omega n \left(\frac{dy}{dt} - \omega lz + \omega nx \right) \\ &\quad + \omega m \left(\frac{dz}{dt} - \omega mx + \omega ly \right) \end{aligned}$$

which, by adding the term $\omega l(\omega lx - \omega lx)$ and remembering that $l^2 + m^2 + n^2 = 1$, reduces to

$$\frac{d^2x}{dt^2} = -2\omega \left(n \frac{dy}{dt} - m \frac{dz}{dt} \right) - \omega^2 x + \omega^2 l(lx + my + nz).$$

Two similar expressions give the acceleration along the Y and Z axes.

The total acceleration experienced by the ion is then

$$\begin{aligned} X &= -\Omega^2 x - \frac{d^2x}{dt^2} = -\Omega^2 x + 2\omega \left(n \frac{dy}{dt} - m \frac{dz}{dt} \right) \\ &\quad + \omega^2 x + \omega^2 l(lx + my + nz) \\ Y &= -\Omega^2 y - \frac{d^2y}{dt^2} = \text{etc.} \\ Z &= -\Omega^2 z - \frac{d^2z}{dt^2} = \text{etc.} \end{aligned} \tag{6}$$

In the case of a magnetic field of force, if the axes of Z be taken parallel to the magnetic field then (l, m, n) become equal to $(0, 0, 1)$ and equations (6) reduce to

$$\left. \begin{aligned} X &= -\Omega^2 x + \omega^2 x + 2\omega \frac{dy}{dt} \\ Y &= -\Omega^2 y + \omega^2 y - 2\omega \frac{dx}{dt} \\ Z &= -\Omega^2 z \end{aligned} \right\}. \tag{7}$$

Now the central force producing the original rotation is $\Omega^2 x$. The perturbing forces are then represented by the terms $\omega^2 x + 2\omega \frac{dy}{dt}$ and $\omega^2 y - 2\omega \frac{dx}{dt}$. Examining these it is seen that $2\omega \frac{dy}{dt}$ and $-2\omega \frac{dx}{dt}$ are the X and Y components of a force $2\omega v$ acting perpendicularly to v , the linear velocity of the ion. If then a charged ion move in a magnetic field with a velocity v , $2\omega v$ is the force it would experience due to the magnetic field. The terms $\omega^2 x$ and $\omega^2 y$ represent centrifugal forces due to the impressed velocity ω , and in the first approximation may be neglected. Finally, if $K = 2\omega$, the above equations become identical with those of Lorentz.

Equations (6) and (7) are sufficiently general to cover all hitherto observed phenomena. Thus to explain the case where

the central line of the triplet (line 2, Fig. 2) is doubled, it is only necessary to write the equation for Z in the form $Z = A \sin \Omega t$ where A is a periodic function of t of the form $A = a \sin nt$. Substituting this in the equation $\frac{d^2z}{dt^2} = -\Omega^2 z$ and integrating we get

$$Z = a \sin nt \sin \Omega t = a/2 [\cos(\Omega - n)t - \cos(\Omega + n)t],$$

which represents two vibrations of frequency

$$(\Omega \pm n)/2\pi. \quad (8)$$

The case of reversed polarization may be covered by supposing the value of n to be such as to separate the components sufficiently to place them outside the other lines of the triplet. In a similar manner the doubling of the outer members of the triplet may be accounted for which would also cover the case of a quartet when viewed parallel to the magnetic field. In this way all the various cases of multiple lines are satisfactorily explained.

Reverting to equations (2) and (3) we see that the change of period of the ion is expressed by the equation,

$$T - T' = \frac{2\pi\sqrt{m}}{K} \cdot \frac{eH}{2K\sqrt{m}}.$$

The proportional change will then be

$$\frac{T - T'}{T} = \frac{eH}{2K\sqrt{m}} = \frac{eHT}{m4\pi}$$

since

$$K = \frac{2\pi\sqrt{m}}{T}.$$

Finally, since $T = \frac{\lambda}{v}$ and $T' = \frac{\lambda'}{v}$ where v = velocity of light, this

becomes

$$\frac{\lambda - \lambda'}{\lambda^2} = \frac{eH}{m4\pi v}. \quad (9)$$

Where λ = wave-length of the spectral line for zero magnetic field.

λ' = wave-length of the spectral line with magnetic field.

v = velocity of light, 300,000,000 cm.

H = intensity of magnetic field in C. G. S. units.

Equation (9) may be written

$$\frac{e}{m} = \frac{\lambda - \lambda'}{\lambda^2} \frac{4\pi v}{H} \quad (10)$$

and it is seen that a measurement of the change of wave-length enables us to determine the ratio of the ionic *charge* to the ionic *mass*.

Zeeman¹ finds for this ratio, in the case of the blue line of Cd ($\lambda = 4800$) 24×10^6 , while Preston² has found that this value must be determined separately for each line, but that a possible classification of lines may be made similar to the chemical classification of Keyser and Runge.³

Another interesting and significant observation is that of Ames⁴ et. alt. who find that in the case of some iron lines, there appears to be no magnetic effect, while those lines which show the greatest magnetic shift are the ones which show the greatest pressure shift, and those which show but little magnetic shift are the ones of little pressure⁵ shift.

The present status of the subject may be summarized as follows:

1. In general, spectral lines are influenced by the magnetic field when the radiations emanate from a source of light in the magnetic field.⁶ The magnetized system of lines is symmetrical with respect to the original unmagnetized line. New spectral lines may be produced by the magnetic field.⁷

2. Viewed parallel to the magnetic field the spectral line is, in general, doubled, but it may become a single or multiple line. In the case of a single line there is no polarization;⁸ in all other cases the components are circularly polarized; the shorter wave-length in the direction of the magnetizing current, the longer wave-length in the opposite sense.

3. Viewed in a direction perpendicular to the magnetic field the

¹ Acad. Amsterdam, 1897-1898, p. 260.

² Phil. Mag. (5), 45, p. 337.

³ Wied. Ann., Bd. XLIII., p. 394, 1891.

⁴ Astro. Phys. Jr., 8, p. 50, 1898.

⁵ Astro. Phys. Jr., 6, p. 169, 1897.

⁶ Astro. Phys. Jr., 8, p. 49.

⁷ Astro. Phys. Jr., 9, p. 47.

⁸ This would be true of the lines seen by Becquerel and Ames, showing reversed polarization, when viewed parallel to the magnetic field.

outer components are plane polarized so that their vibrations are perpendicular¹ to the magnetic field ; the inner components having their vibrations parallel to the magnetic field.

4. The general form of the line viewed perpendicularly to the magnetic field is a triplet, but it is sometimes of complex structure.

5. The amount of magnetic action, measured in change of period of the spectral line, is not a simple function of the wave-length ; nor is it a constant for all wave-lengths ; nor a constant for all lines of a given substance ; nor is it a simple function of wave-length for the lines of a given substance.

6. The magnetic action is proportional to the field strength (being limited, however, by temperature and pressure) ; and there appears to be a possible classification following the chemical classification of Mendelejeff and of Kayser and Runge.

PART TWO. EXPERIMENTAL WORK.

Introductory Note.—From the foregoing survey of the subject we are led to believe that a comprehensive study of the problem consists essentially of two parts :

1. A qualitative analysis of as many spectral lines, emanating from as many different substances as possible, with a classification according to the type of line produced.

2. Quantitative measurements of the change of wave-length and of the ratio e/m , and a classification based upon these measurements.

All spectral lines belonging to the same group in both classifications may then be regarded as possessing related properties, and the ratio e/m as determined from such a group of lines should have the same value.

Such a series of observations would as Preston remarks “ afford a valuable means of inquiry into the so far hidden nature * * * of the radiation from a luminous body, and also give us some clearer insight into the structure of matter itself.”

Outline of Work.—In the following research the complete study of the subject was not attempted, this being manifestly too great a task for the limited time available. The preliminary ground has, however, been cleared and a beginning made.

¹ Exception, see Astro. Phys. Jr., 8, p. 50, 1898.

The work was subdivided as follows :

Section I. A preliminary survey of the field with a view of determining the conditions limiting the observation of the magnetic phenomenon.

Section II. A comparison of the ease of manipulation and range of the two methods outlined above.

Section III. To ascertain whether the magnetic effect is radically different at different temperatures.

Section IV. To measure the magnetic shift of as many lines as the time available would permit; studying, also, the state of polarization of the components.

SECTION I. A PRELIMINARY SURVEY TO DETERMINE THE CONDITIONS LIMITING THE OBSERVATION OF THE MAGNETIC PHENOMENA.

APPARATUS. I. *Magnet*.—This was of the usual upright type; the base, cores, coils, pole heads, and cores to pole heads are all separable; the base and cores to the pole heads are of mild steel, the rest of the magnetic circuit being made of that form of cast iron known as *Mitis Metal*. An elevation of the magnet is shown in Fig. 4, drawn to $\frac{1}{8}$ scale.

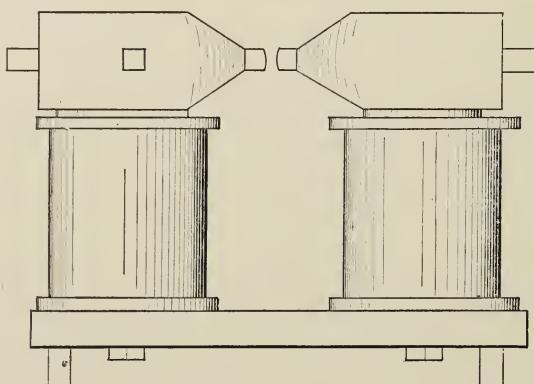


Fig. 4.

The cores to the pole heads are one inch in diameter, and it soon became apparent that they, together with the mass of iron behind them, tended to lower the temperature of the flame. A second

pair of cores was made of the form shown in Fig. 5. This form also concentrated the magnetic field, thereby increasing its strength. A third core was prepared similar to those shown in Fig. 4, with a $\frac{1}{4}$ -inch hole throughout its length; this core was for use in viewing the flame parallel to the magnetic field.

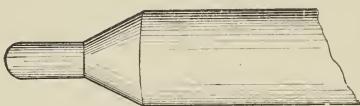


Fig. 5.

II. The Flame.—The flame of a small Bunsen burner was first used but was found to be too large. A small blast lamp was then made of glass, and a foot bellows used. This gave a small conical jet of flame about 3 inches high when the blast was inactive, and about $\frac{3}{4}$ inch high with the blast. This form of lamp, shown in Fig. 6, proved highly satisfactory.¹

To color the flame a strip of asbestos wick supported by platinum wire was first used. A bead of fused sodium carbonate was also

used, and, finally, a rod of sodium glass was adopted as needing the least attention. When a very bright flame is used recourse to the fused bead may be had, but such a flame generally gives rise to spontaneous reversals. In the latter experiments the glass lamp was set aside for the burner of an oxy-hydrogen lamp, and the foot-blast was replaced by an oxygen tank.

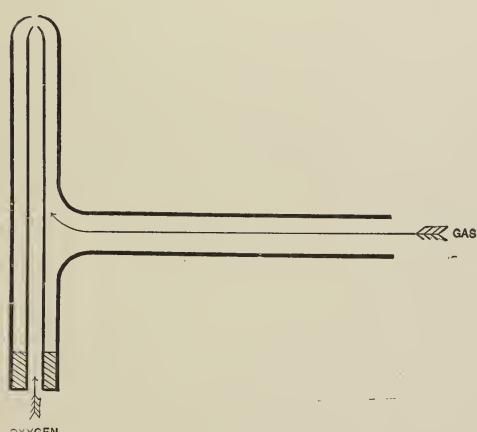


Fig. 6.

III. Dispersion Apparatus.—The first trials were made with a plane grating spectroscope; the higher spectra were especially dim. No success was attained with this apparatus. Next a Rowland concavé grating of 14,436 lines to the inch and five feet focal distance was mounted at one end of the table, the magnet being at the other end. The spectrum was viewed with a telescope. It was

¹ In time the glass about the opening cracks away but the whole lamp is easy of construction.

with this mounting and with an oxygen gas flame that the first results confirmatory of Zeeman's work were obtained.

The arrangement of apparatus is shown in Fig. 7. The lines D_1 , D_2 were very sharp and bright. When the current was turned

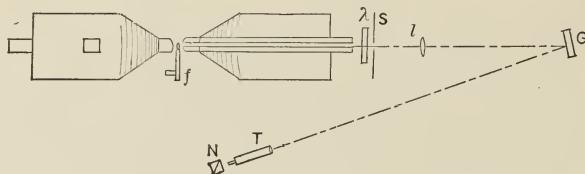


Fig. 7.

on each line grew broad and after a few seconds became distinctly double, a sharp dark line separating the components throughout their whole length. The lines were visibly brightened, as was also the whole flame.

When the current was interrupted the lines appeared to collapse, and after a second or so would again become sharp. Frequently the doubling of the lines would persist for a few seconds after the current was broken and the dark dividing line could be seen, though narrower than with the full field.

For the doubling a field strength of about 18,000 C. G. S. units was used, with weaker fields only a broadening of the line could be observed.

Polarization.—With zero field the light was found to be slightly polarized by reflection from the grating. The polarization with

full field is represented in Fig. 8, the accelerated components D_1^2 , D_2^2 harmonize in their sense of rotation with the magnetizing current. The $\frac{1}{4}\lambda$ plate reduces the circular polarization to plane polarization.



Fig. 8.



Fig. 9.

Viewing the flame in a plane perpendicular to the magnetic field the pole cores shown in Fig. 5 were used. The tripling can best be observed by means of the nicol; without the nicol the tripling could be faintly seen, but the strength of field necessary and the instrumental difficulties present make the

observation of the phenomena far from satisfactory. The planes of vibration of the components are shown in Fig. 9. As no quarter wave plate is used this is the state of vibration in the ray of light itself.

It being apparent that no measurements could be taken without resorting to photography, the study was concluded at this point and work with the interferometer begun.

Summary. The following points may be noted as covering the first point aimed at in the work :

1. The magnetic separation of the sodium lines D_1 , D_2 , as given by a naked flame, cannot be distinctly observed at the temperature of the Bunsen flame, nor of the air-blast flame, nor even at the temperature of the oxygen gas flame, unless precautions are taken against spontaneous reversals.
2. The phenomenon can be much more satisfactorily observed parallel to the magnetic field when perpendicular to it, as the strength of field necessary to produce a pure (or visual) triplet is twice that necessary to produce the doublet.
3. There is a very perceptible time lag both when the magnet is excited and when the current is broken, during which period the lines show an inertia effect. This lag does not seem to be wholly due to the self-induction of the fields, but may be partially visual and partially *ionic*.
4. A field strength of at least 15,000 C. G. S. units seems to be necessary for satisfactory observation.
5. Spectra above the second order are too faint for good effects.

PART TWO. THE INTERFEROMETER METHOD.

Professor Michelson¹ has shown the peculiar adaptability of the interferometer for the class of research here considered.

The instrument used in the present case was received March 1, 1898, and has proved highly satisfactory for the work undertaken.

The instrumental difficulties attending the use of the Interferometer are such as perhaps to warrant a word about them. These difficulties arise from two sources : (1) defective workmanship in the instrument, and (2) those due to the observer. These will be considered in order : (1) the most serious error that can be present is a lack of parallelism in the ways along which the mirror carriage moves. This is generally manifested by a shifting of the system of fringes in the field of view, and a more rapid loss of visibility on the part of the fringes than should take place.² A second defect is sometimes present in the optical quality of the glass, due either to a defective surface, or to internal stress in the glass. This defect is manifested by a distortion of the fringes which under proper adjustment should appear as circles. A third defect, easily overlooked, is in the silvering of the surfaces. The two end mirrors (see Fig. 10) $M' M''$ should be heavily silvered, and brightly polished. The half-silvered surface (M), however, should have such a film as will transmit and reflect equal amounts of light. If this be true,

¹ Astro. Phys. Jr., 6, p. 48, also 7, p. 131.

² Phil. Mag. (5), 34 p. 286.

50% of the incident light is effective in the two interfering beams, and the fringes are of maximum brightness. There is no direct method of estimating this defect, and experience is the only proper guide. (2) A very considerable amount of patience is necessary to eliminate or minimize personal errors. Fringes are often obtained which appear satisfactory, but from a lack of exact adjustment have a variable focus, are distorted, and rapidly fade out as the movable mirror recedes. If the instrument be in adjustment the fringes appear as concentric circles, and Michelson has shown¹ that if the incident light be parallel then the fringes are at infinity.² If, however, the light be not parallel, then the fringes are in front of the interfering surface (M , Fig. 10), upon it or behind it, according as Δ is greater than, equal to, or less than zero, where Δ is the difference in the paths traversed by the two beams of light. In general it may be said that the incident beam is not parallel, and hence, that the fringes recede as Δ increases. If a telescope is used to view the fringes, the focus will have to be altered slightly, and must soon be adjusted for parallel light. If, on the other hand, the naked eye is used, there is a tendency to focus the eye upon the mirror, instead of upon the fringes. Hence an observer is apt to lose sight of the fringes altogether, or to view them out of focus unless considerable care is exercised upon this point. The real work of the interferometer consists in an eye estimate of the visibility of the fringes as Δ is increased. There is a manifest liability to error in these eye estimates, unless a comparison set of fringes of known intensity is available; but the process in such a case is not only tedious³ but adds to the amount of apparatus to be looked after. On the whole, it is more practicable, and perhaps, as reliable to depend upon the training of the eye that comes from long practice, and in testing the eye from time to time by means of comparison fringes. However, even after a correction⁴ curve is obtained and applied, there is room for a considerable margin of personal error, which is, perhaps, the chief drawback to the method as a whole.

¹ Phil. Mag. (5), 13, p. 239, 1882.

² Except when $\Delta = 0$, when the fringes are on the mirror.

³ Astro. Phys. Jr., 7, p. 133.

⁴ Phil. Mag. (5), 34, p. 283. Such a curve is shown in Fig. 14.

Adjusting the Interferometer.¹

The general disposition of apparatus for the present work is shown in Fig. 10.

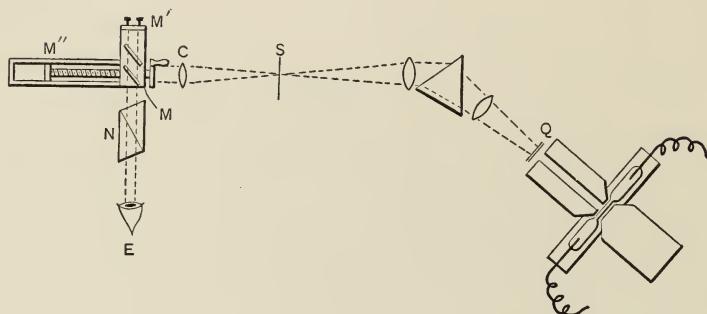


Fig. 10.

M, M', M''. Interferometer mirrors.

Q. $\frac{1}{4} \lambda$ plate.

N. Nicol.

S. Slit.

E. Eye or telescope.

FIRST ADJUSTMENT. To Find the Fringes.—The mirror M' is set so as to be within two or three mm. of the zero of the scale, and a Bunsen flame² colored with a piece of sodium glass is so set with reference to the lens C that the field of the mirrors is uniformly bright. Then a pin, or bit of glass fiber, is fastened by a bit of wax onto the lens, so that its image is in the center of the field. It is preferable to have the image lie obliquely across the field, not horizontally or vertically. The mirror M' is then adjusted until the two images of the pin, made by the two mirrors, M' and M'' , are superposed. When the images lie obliquely the movements of the adjusting screws can be easily followed. When the adjustment is very close, if the room is partially darkened, and the eye focused for a point behind the mirror, the fringes should appear as fine lines covering the field. The eye should now be focused on these fringes, and

¹ The Adjustments of the Interferometer are discussed at length by Professor Wadsworth in PHYS. REV., Vol. 4, p. 480. See also p. 400 of same vol.

² The Bunsen flame occupies the position of slit S in Fig. 10, the rest of the system to the right of S being absent.

the screws adjusted so that the fringes grow broader and more distinct. The final adjustment is reached when they appear as concentric circles, which do not change in appearance, when the eye is shifted in any direction. Under these conditions the two mirrors, M' and M'' , are optically parallel to each other.

SECOND ADJUSTMENT. *To Find Zero of the Instrument.*—The zero of the instrument is the position of zero difference of path between the two interfering beams of light. It is found by obtaining fringes with white light and adjusting on the central black fringe of this system. To obtain fringes with white light the mirror M' should first be adjusted so as to give the fringes as vertical lines of small curvature. If now the mirror M'' be moved back and forth by slow motion, the fringes will be seen to change curvature at one region. The exact point of this change of curvature is the point sought. Having adjusted approximately for this, a candle flame should be set before the Bunsen flame, giving the white image of the candle flame with a background of sodium fringes. The mirror M' may now be slowly moved by means of the tangent screw, the motion being followed by the eye, in the progression of the (faint) sodium fringes. When the zero point is approached brilliant chromatic fringes, from ten to twenty in number, will appear in the image of the candle flame. If now the candle be removed, and the air cut off from the Bunsen flame the chromatic fringes are very distinct and brilliant.

The zero point will shift a little with change of temperature, and still more by a change in the distance of the mirror M' from M due to difference in the manipulation of the adjusting screws. The zero should therefore be observed before each set of observations.

Visibility Curves.

The theory of the visibility curve has been given by Michelson, and its adaptation shown to problems where the distribution of light as a source of radiation is to be determined.

The following curves were taken using sodium light as source of illumination, and viewing the light first perpendicular to the field, and than parallel to it.

I. Perpendicular to the Field. The curves taken are shown in Fig. 11, *A*, and the corresponding distribution of light in Fig. 11, *B*.¹

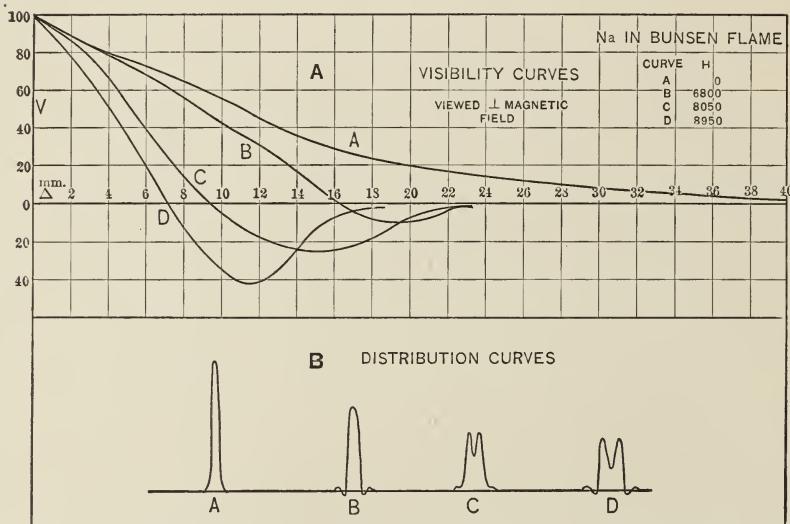


Fig. 11.

In this curve the nicol prism was not used, and the loss of light by the central component (which is known to be present), is such as to render the line an apparent double.² In the curves *B*, *C*, *D* the main components seem accompanied by small companions some having negative ordinates. Now since this would indicate negative intensities, they indicate errors in observing the curves *A*. The analysis of the curves *A* then furnish a valuable check upon their correctness.

II. Light Viewed Parallel to the Magnetic Field. Curves were taken for this position both with and without the nicol. The results are shown on Figs. 12 and 13. These are substantially identical and show first a broadening and then a doubling of the spectral lines. The presence of the negative ordinates is also seen and yet the substantial results are clear.

¹ Acknowledgment is due to Professor Michelson who kindly facilitated the analysis of these curves on the Harmonic Analyzer at the University of Chicago.

² See part I, also Michelson, Astro. Phys. Jr., 6, p. 49.

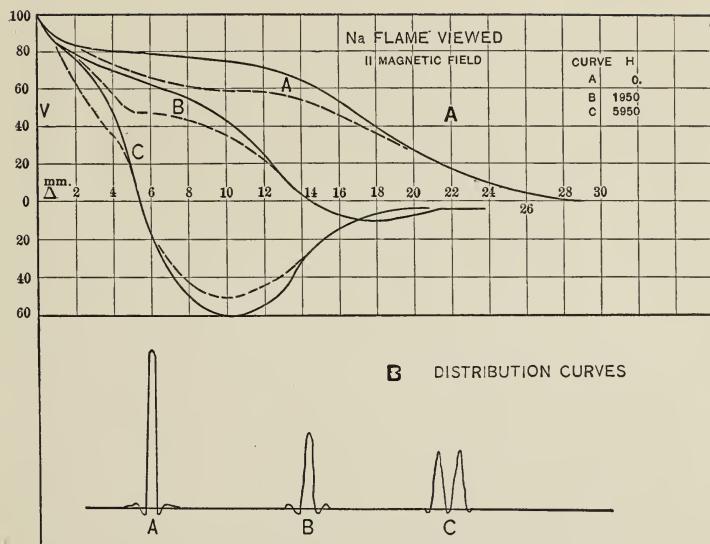


Fig. 12.

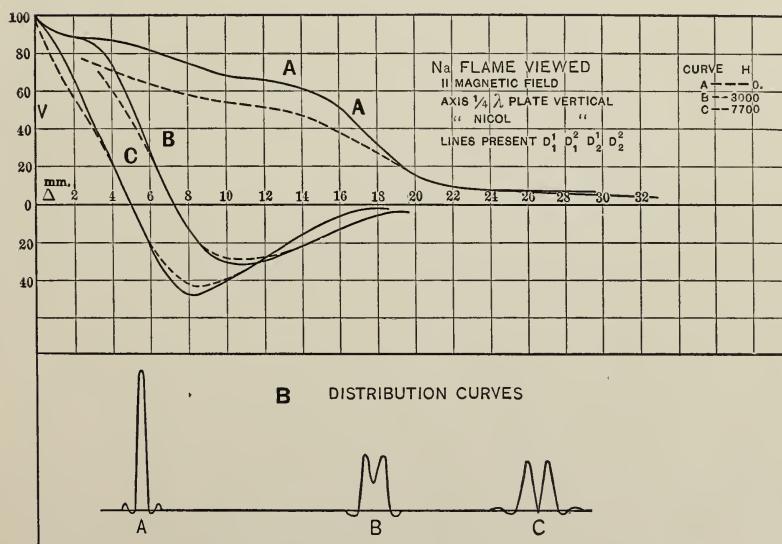


Fig. 13.

Having analyzed these curves a comparison curve was then taken to observe what correction was necessary to reduce the eye estimates shown in the full line curves of Figs. 11, 12 and 13. The resulting curve is shown in Fig. 14 and the corrected visibility curves are shown in the dotted curves of Figs. 12 and 13. These corrected curves were not analyzed, but their distribution curves would be more nearly correct than those shown. We have then in the curves shown all observational errors present; the result, though not satisfactory, is by no means bad. This form of error is very hard to avoid and long practice alone can eliminate it.

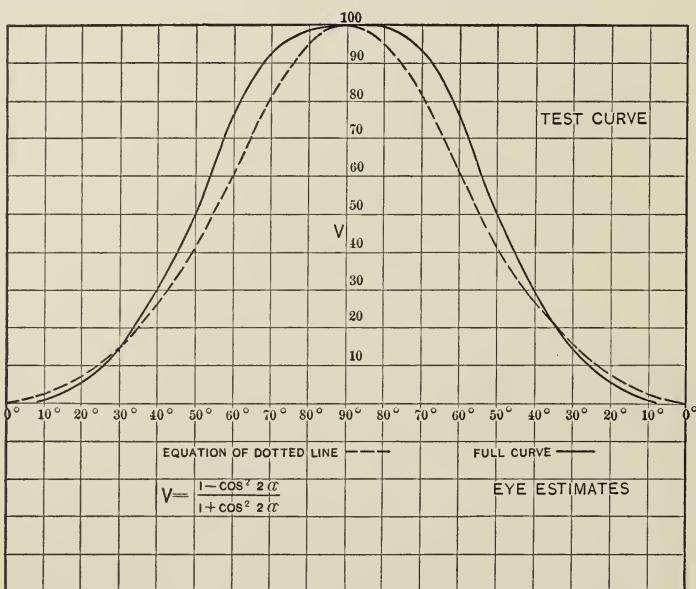


Fig. 14.

Polarization.—The interferometer was found to be capable of showing the state of polarization in the most elegant manner, not only identifying the plane of polarization, but also immediately identifying the accelerated from the retarded ray.

Thus with the axis of the $\frac{1}{4}\lambda$ plate vertical the nicol was rotated and the action and character of the fringes observed. The following were the results obtained :

TABLE II.

Position of Axis of Nicol.	Action of Fringes on Magnetization.	Character of Central Fringe with Magnetic Field.
0° Vertical.	Become hazy.	Hazy.
+ 45°	Expand and remain distinct.	Distinct, dark.
+ 90° Horizontal.	Become hazy.	Hazy.
+ 135°	Contract and remain distinct.	Distinct, light.
180° Vertical.	Become hazy.	Hazy.
- 135°	Expand and remain distinct.	Distinct, dark.
- 90° Horizontal.	Become hazy.	Hazy.
- 45°	Contract and remain distinct.	Distinct, light.
0° Vertical.	Become hazy.	Hazy.

It is evident that the haziness at $0^\circ \pm 90^\circ$ and 180° is due to the simultaneous contracting and expanding of the fringes. Further analysis shows us that when the fringes expand the retarded component is present and when the fringes contract the accelerated (or shorter wave-length) component is present.

This is shown in Fig. 15. Unpolarized light is represented by the superposition of the two perpendicular planes of vibration.

From this general survey of the interferometer we derive the following generalizations :

1. The interferometer is capable of showing the magnetic effect for field strengths below 1,000 C. G. S.
2. The visibility curves, even under unfavorable circumstances, show clearly the general character of the magnetic effect and when checked through a long series, by means of an harmonic analyzer, furnish an incomparable method of analysis.
3. When unaccompanied by such a check errors are not readily eliminated and for quantitative measurements of change in wavelength another use of the interferometer furnishes a better method.

Inasmuch as the visibility curve analysis was, not under the circumstances, entirely available, attention was turned to the development of point 3 above and carried out as follows :

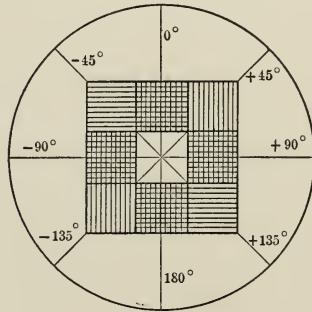


Fig. 15.
Axis of $\frac{1}{4} \lambda$ plate vertical,
Nicol rotated.

Measurements of Change of Wave-length.

Method.—Professor Michelson¹ in determining the difference in wave-length between the components of the magnetic line makes use of what he calls the “period of coincidence due to doubling.” This may be defined as follows: Consider two systems of fringes produced by light sources differing but little in wave-length. Then the *resultant* system due to both sources, will be the resultant of the overlapping of the component systems, and at certain points destructive interference will take place, and at others reinforcement. The fringes will then run through a series of maxima and minima as Δ is increased. The distance from maximum to maximum is the “period.” If Δ be measured from “zero” to the first point of reinforcement, then the following equation holds

$$\Delta_p = N\lambda = (N + 1) \lambda' \quad (11)$$

where N is the total number of fringes; and Δ and λ are measured in mm. In the case of the lines $D_1 D_2$ these maxima occur every 988 fringes and $\Delta_p = 0.58242$ mm. In the case, however, of the magnetic shift the difference of wave-length is so small that Michelson finds the values of Δ to range from 58 mm. to 14 mm. as the magnetic field increases to 4,000 C. G. S. units. The same method is also described by Perot and Fabry² and is especially applicable where the difference of wave-length is sufficient to give a difference of color to the two sets of fringes.

When applied to the magnetic components the method is open to improvement. In the first place the fringes, at a point where $\Delta = 50$, or even 30, are so narrow and frequently so faint that an accurate determination of points of reinforcement (*i. e.*, where both systems agree in phase) is not easy. Secondly, since the magnetized and unmagnetized fringes cannot be simultaneously observed, the above determination consists in finding the value of Δ at which the closing of the magnet shifts the system the double width of one fringe. This determination for high values of Δ is not as easy as might be wished.

¹ Astro. Phys. Jr., 6, p. 50.

² Astro. Phys. Jr., Feb., 1899. See Appendix III.

There are, however, two possible modifications. If the nicol be not used, it has been seen that the magnetized components draw apart, one being retarded and the other accelerated. The corresponding effect on the fringes is to cause the original system simultaneously to contract and expand. If now the point is found at which the two systems of fringes are tangent to each other, each fringe will have shifted $\frac{1}{2}$ its own width, showing that the accelerated system is $\frac{1}{2}$ period in advance of the retarded one, or $\frac{1}{4}$ period removed from the original system. We then have the equation

$$\Delta_{\frac{1}{4}p} = N\lambda = (N + \frac{1}{4})\lambda'. \quad (12)$$

The second modification consists in using the nicol and in thus quenching one of the component systems. Now concentrate the attention upon one fringe, either adjusting a pointer (fastened to lens c , Fig. 10) to its edge, setting the fringe tangent to a line drawn on the mirror M , or by merely observing the central fringe of the system. Let it be supposed that the central fringe is dark, and the field unmagnetized. On turning on the current the system contracts, and for a certain value of Δ will be the exact complement of the first, the center being now light instead of dark.

Under these conditions the fringes have shifted over their own width, and the following equation is obtained.

$$\Delta_{\frac{1}{2}p} = N\lambda = (N + \frac{1}{2})\lambda'. \quad (13)$$

Still a third modification would be as follows: Let the field remain magnetized and set the fringe, with the nicol in a given position, tangent to the fixed line. Then let the nicol be rotated 90° , so as to bring the other system of fringes into view; then that value of Δ which renders the line a common tangent to both systems gives a difference of period such that

$$\Delta = N\lambda_1 = (N + 1)\lambda_2$$

where λ_1 and λ_2 are the component lines. Also since λ_1 and λ_2 lie symmetrically with respect to λ (the unmagnetized line) we have

$$\Delta = N\lambda_1 = (N + \frac{1}{2})\lambda = (N + 1)\lambda_2$$

or, putting the equation in terms of λ ,

$$\Delta = N\lambda = (N \pm \frac{1}{2})\lambda' \quad (14)$$

which is identical with equation (13).

Since in the above equations N is the total number of fringes passed over from the position $\Delta = 0$, some means must be had of knowing its value. This is obtained very simply as follows: If λ be given in mm. then $1/\lambda$ equals the number of fringes per mm., and $N = \Delta/\lambda$, Δ being measured in mm. Substituting this value in the above equations and solving for $\lambda - \lambda'$, the following equations are obtained

$$\left. \begin{array}{l} \text{For } \Delta_p \text{ equation (11)} \\ \lambda - \lambda' = \frac{\lambda\lambda'}{\Delta} = \frac{\lambda^2}{\Delta} \quad (11') \\ \\ \text{For } \Delta_{\frac{1}{4}p} \text{ equation (12)} \\ \lambda - \lambda' = \frac{\lambda\lambda'}{4\Delta} = \frac{\lambda^2}{4\Delta} \quad (12') \\ \\ \text{For } \Delta_{\frac{1}{2}p} \text{ equation (13)} \\ \lambda - \lambda' = \frac{\lambda\lambda'}{2\Delta} = \frac{\lambda^2}{2\Delta} \quad (13') \end{array} \right\} \quad (15)$$

For a given, value of $\lambda - \lambda'$ we have the relation

$$\Delta_p = 4\frac{1}{4}p = 2\Delta_{\frac{1}{2}p}. \quad (16)$$

This last equation would indicate that the second method given by equation (12') would be the most accurate since it gives the smallest values of Δ and hence the widest and brightest fringes. It must, however, be noticed that the condition expressed by equation (12) is that the two systems of fringes shall differ by $\frac{1}{2}$ period and hence that the dark rings of one system shall coincide with the light rings of the other. Hence the field will be uniformly illuminated and the fringes disappear. In practice there is found to be a region over which the fringes are blurred and the exact point of extinction is difficult to determine.

On the other hand, in the third method the fringes are always sharp and for values of H above 2,000 C. G. S. units the values of Δ are small enough to render the fringes sufficiently wide for satisfactory observation.

With a little practice the method of observing the central fringe was found to give concordant readings and hence was used in preference to setting the fringe tangent to a line.

A comparison of the three methods is given by equation (16). Still further light is given by substituting the value of $\frac{\lambda - \lambda'}{\lambda^2}$ from equations (13) in equations (9).

Making this substitution we get

$$\left. \begin{array}{ll} \text{1st method.} & H\Delta = 4\pi v \frac{m}{c} \dots (11'') \\ \text{2d} & " \quad H\Delta = \pi v \frac{m}{e} \dots (12'') \\ \text{3d} & " \quad H\Delta = 2\pi v \frac{m}{e} \dots (13'') \end{array} \right\} \quad (17)$$

If m/e be regarded as constant, these equations all represent rectangular hyperbolæ asymptotic to H and Δ taken as axes.

It is thus seen that curves expressing the relation of H and Δ furnish a ready means of comparison both by equations (16), and by their form as indicated by equations (17). Such a set of comparison curves was taken with an oxygen gas flame colored with sodium. In Table III. the values of $H\Delta$ for different parts of the curve are given, and the curves themselves are given in Fig. 16.

TABLE III.

Method II.			Method III.		Method I.	
H^a	Δ	$H\Delta$ (curve A.)	Δ	$H\Delta$ (curve B.)	Δ	$H\Delta$ (curve C.)
2,300	15.4	354	21.4	492	13.1	300
3,000	12.0	360	16.7	500	12.0	360
4,000	9.0	360	13.3	530	11.0	440
5,000	7.3	365	10.5	526	10.0	500
6,000	6.2	370	8.2	490	9.0	540
7,000	5.5	385	6.8	475	8.2	575
8,000	4.8	384	5.8	465	7.4	590
9,000	4.3	386	5.2	470	6.7	602
10,000	3.8	380	4.8	480	6.1	610
11,000	3.5	386	4.5	496	5.6	615
12,000	3.4	408	4.4	528	5.3	635

^a H throughout these experiments was determined by the ballistic method.

Curves *A* and *B* corresponding to Methods II. and III. show a close correspondence with equation (17) while curve *C* is manifestly unreliable.

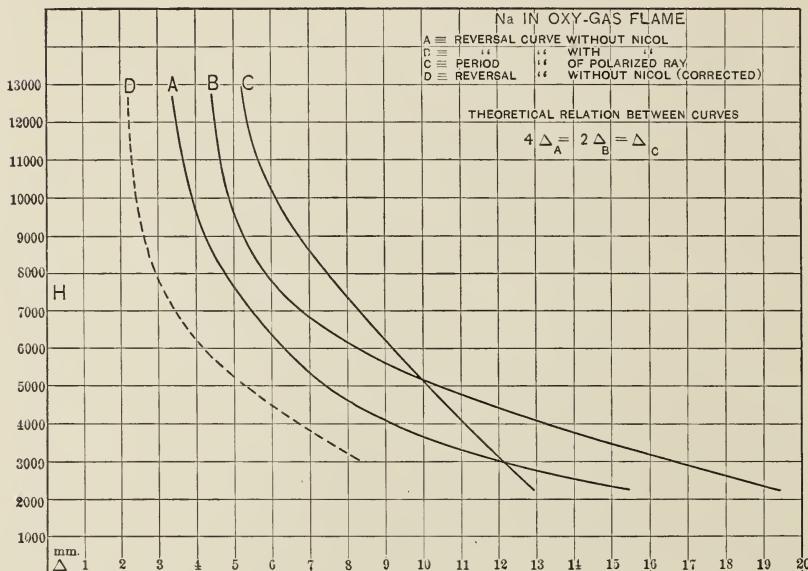


Fig. 16.

Comparing curves *A* and *B* we see that equation (16) is not fulfilled, and hence that either one or both curves are displaced. From the fact already pointed out as to the difficulty of making settings for curve *A* we have no hesitation in concluding that the readings for this curve are uniformly too high.¹ Assuming that curve *B* is correct, the dotted curve *D* gives the corresponding position for curve *A*.

SECTION III. COMPARISON OF MAGNETIC SHIFT AT DIFFERENT TEMPERATURES.

To investigate this the sodium flame was used and reversal curves taken, by Method III., under the following conditions : I. Bunsen flame, II. Oxy-gas flame, III. Vacuum tube.

¹ It is not to be supposed that curve *A* represents the greatest accuracy attainable by Method II. Continued observation would undoubtedly render this method quite available.

The light was viewed parallel to the magnetic field as giving the most uniform structure of line (see Fig. 3, *B*), and a $\frac{1}{4}\lambda$ mica plate used. The following tables are selected as representative.

TABLE IV.
Reversal Curves. Na in Bunsen Flame.

<i>H</i> C. G. S. units.	Δ mm.	$\lambda - \lambda'$ A. U.
0,000	∞	0.000
3,000	12.80	0.135
4,000	11.65	0.155
5,000	10.60	0.168
6,000	9.60	0.180
7,000	8.50	0.205
8,000	7.70	0.230
9,000	6.40	0.274
9,500	5.40	0.325
9,600	5.00	0.350
9,700	4.90	0.370
9,800	4.80	0.375
10,000	4.70	0.380
11,000	4.65	0.380
12,000	4.65	0.380
13,000	4.65	0.380

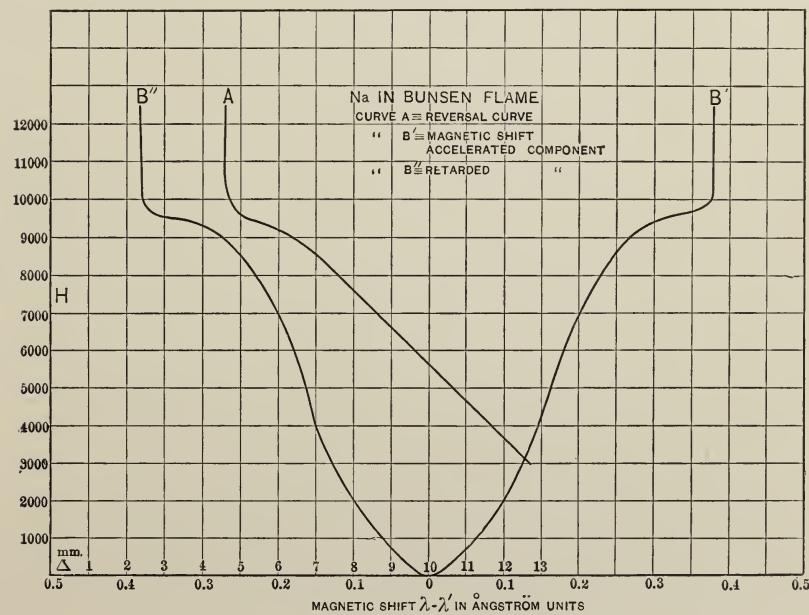


Fig. 17.

Table IV. is platted in Fig. 17. The part of the curve lying between 8,000 C. G. S. and 11,000 was checked up with special care and the knee in the curve verified. For all readings where Δ is less than 3 mm. a small change in Δ indicates a large change in $\lambda - \lambda'$, and hence the readings become less sensitive. The departure of the reversal curve from the normal hyperbolic form would indicate, as is shown in curve *B*, Fig. 17, an ionic inertia or lag, due either to low temperature or high density of the gas, or to both. This constraint retards the change in wave-length but suddenly gives way at a field strength of about 9,500. Above a value of H , of 10,000 the mean wave-length of the lines do not seem further to change, increase in H beyond this point being effective in broadening the lines rather than in separating them.

In curves *A* and *B* (Fig. 18) a small flame 2 cm. high was used with the tip of the inner cone opposite the aperture. In curve *C* the jet was 15 cm. high and the base of the flame opposite the opening. The flame in both cases was colored with a bead of fused sodium carbonate.

Curve *C* shows the same "lag" as does the Bunsen flame, and the maximum value of $\lambda - \lambda'$ is not so large. In curves *A* and *B*,

TABLE V. See Fig. 18.

Reversal Curves. Na in Oxygen-gas Flame.

H	Δ_A mm.	Δ_B mm.	Δ_C mm.	$(\lambda - \lambda')_A$	$(\lambda - \lambda')_B$	$(\lambda + \lambda')_C$
0	∞	∞	∞	0.000	0.000	0.000
2,500	17.20	18.50	19.40	0.105	0.097	0.090
3,000	16.00	17.15	18.00	0.111	0.105	0.100
4,000	13.30	14.35	15.35	0.135	0.125	0.115
5,000	10.80	11.70	12.80	0.164	0.151	0.140
6,000	8.25	9.00	10.70	0.212	0.197	0.165
7,000	6.20	6.95	9.10	0.285	0.251	0.197
8,000	5.10	5.80	7.80	0.345	0.304	0.228
9,000	4.55	5.10	6.90	0.390	0.345	0.254
9,500	4.40	4.90	6.50	0.400	0.360	0.275
10,000	4.30	4.70	6.00	0.410	0.375	0.310
10,500	4.25	4.50	5.40	0.420	0.402	0.334
11,000	4.15	4.40	5.15	0.430	0.402	0.345
12,000	4.05	4.40	5.10	0.442	0.402	0.345
12,700	4.00	4.40	5.10	0.442	0.402	0.345

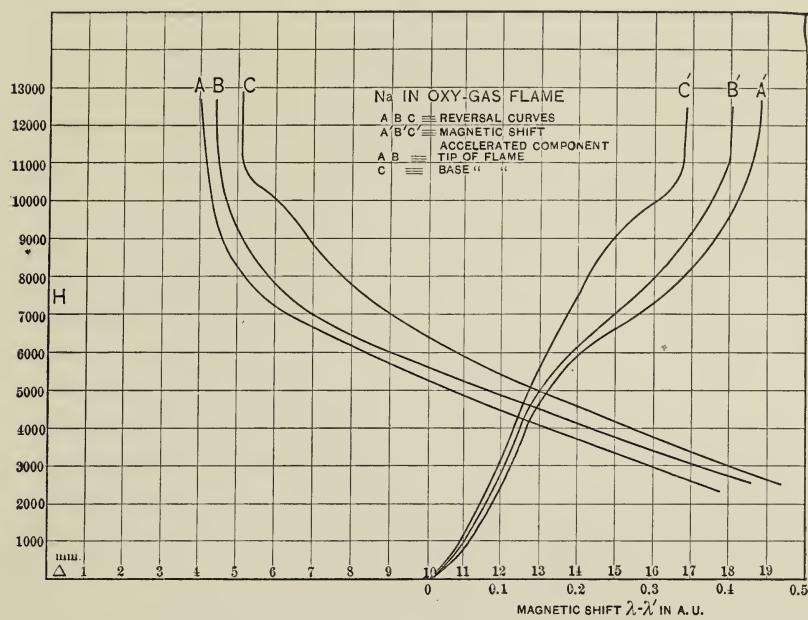


Fig. 18.

which are at a higher temperature, this lag is overcome at a lower value of H , and the maximum value of $\lambda - \lambda'$ is larger.

TABLE VI. See Fig. 19.
Reversal Curve. Na in Vacuum Tube.

H	Δ mm.	$\lambda - \lambda'$ A. U.
0	∞	0.000
5,000	11.20	0.160
6,000	9.50	0.186
7,000	7.90	0.225
8,000	6.30	0.280
9,000	5.00	0.352
9,500	4.60	0.384
10,000	4.25	0.415
10,500	4.10	0.431
11,000	4.00	0.441
12,000	4.00	0.441
13,000	4.00	0.441

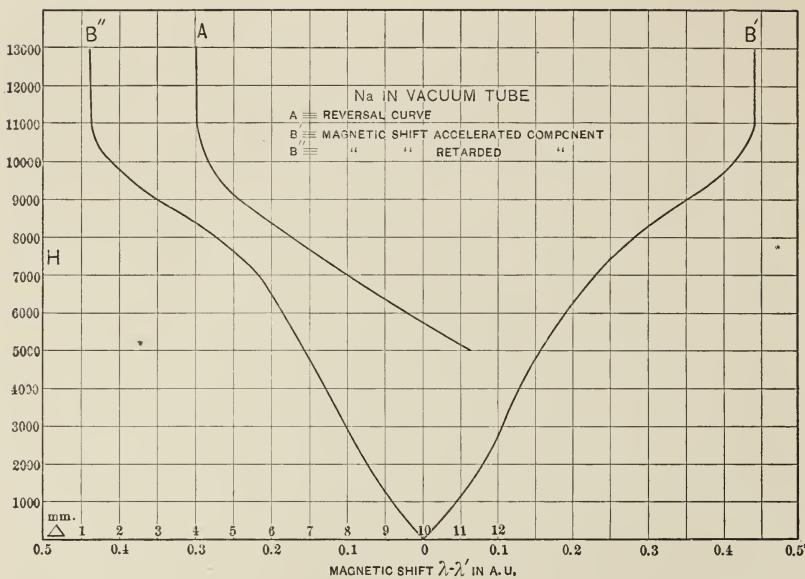


Fig. 19.

Fig. 19 would seem to show that at the temperature of the vacuum tube the change of $\lambda - \lambda'$ is nearly proportional to H up to 7,000 C. G. S., at which point the "lag" is overcome. At 11,000 the maximum for $\lambda - \lambda'$ is reached.

In the case of the vacuum tube, the tube was arranged as shown in Fig. 20. The brass tube T is made to fit over the pole heads of the magnet, and is capped at either end, thus forming a metal box. Into this box the vacuum tube is placed, being held from contact with the metal by asbestos wool.

A Bunsen flame is so supported as to play horizontally upon the tube. Small glass tubes enter the caps and convey wires to the terminals of the vacuum tube which was energized by an induction coil of the usual type. The substance whose spectrum was to be examined was placed in the tube, in the form of filings, and the tube then exhausted to between 1 mm. and 2 mm. pressure. The brass tube was then heated by means of the

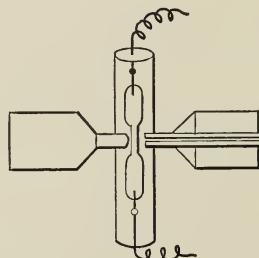


Fig. 20.

Bunsen flame, until the spectral lines appeared sharp and bright. In some cases the heating was pushed too far, resulting in chemical action on the glass.

Upon using the vacuum tubes it was found that the glass of the tube itself acted as a retardation plate, adding its effect to that of the $\frac{1}{4} \lambda$ mica plate. It was thus found impossible to produce an exact retardation of $\frac{1}{4}$ period. With reversed positions of the nicol prism, however, the two systems of fringes were found to be clear and sharp so that the arrangement was satisfactory if not ideal. The mica plate was used part of the time, and part of the time discarded.

In the preceding curves the effect of pressure cannot be separated from the temperature effect. The combined effect is what is observed. An interesting detail yet to be studied is the case where the pressure is gradually varied, and a pressure curve obtained.

Summary.—From a study of these curves we draw the following conclusions with regard to the effect of temperature.

I. At the temperature of the Bunsen flame there is a distinct ionic "lag" or constraint which is overcome, suddenly, at a field strength of 9,500 C. G. S. units.

II. This ionic lag becomes less as the temperature rises and is practically absent at the hottest temperature of the oxygen-gas flame or of the vacuum tube.

III. The change in wave-length reaches a maximum value depending upon the temperature (and pressure); the maximum is reached at about 11,000 C. G. S. For values of H above this the magnetic effect is to broaden the lines and not to separate them further.¹

This latter point receives support from Michelson's echelon spectroscope which shows the sodium lines to belong to type II. (See Fig. 3, *B*.)

NOTE. An attempt was made to take observations with the spark gap. A glass tube 1 mm. bore was taken, and metallic sodium crowded into it until 2 or 3 cm. of the tube at the center was filled. The tube was then cut at the middle of the sodium, and the free ends used as spark terminals.

On turning on the magnet the electro-dynamic action of the field caused a spark to fan out showing in a very pretty manner the magnetic equipotential surfaces. This has also been observed by M. Cornu. (Astro. Phys. Jr., 7, p. 166, note 2.)

¹ This may be peculiar to the sodium lines, and be due to the simultaneous presence of the lines D_1 and D_2 . No observations have yet been made upon them separately, the dispersion necessary to isolate them being too great.

SECTION IV. MEASUREMENTS OF MAGNETIC SHIFT.

Equation (9) may be put into the form

$$\lambda - \lambda' = \left(e/m \frac{\lambda^2}{4\pi v} \right) H \quad (18)$$

showing that for a given value of λ the change in wave-length should be proportional to H provided no constraint is present. We have seen that in the case of sodium (and presumably for all substances) a constraint is present at low temperatures, but that it disappears as the temperature rises. In the measurement of magnetic shift it is important that the temperature be high, and, preferably, that the pressure be low. These conditions are fulfilled by the vacuum tube; hence it was used as a source of illumination.

It was primarily intended to make an extended series of observations upon the spectral lines of different substances, but the difficulties encountered of breaking tubes, deposits in the capillary, and chemical action within the tube, were such as to reduce the number of satisfactory observations to sodium, zinc, mercury and cadmium. Difficulty was also met with in getting readings upon lines lying near either end of the visible spectrum.

The following are the data taken:

Sodium. *Yellow lines ($D_1 D_2$)*.—The probable linear relation of $\lambda - \lambda'$ to H can be derived from Fig. 21, curve *B*, by drawing a line from the origin tangent to the curve. This is given in Fig. 24.

Zinc. *Blue line.* $\lambda = 4810.724$. (For arrangement of apparatus, see Fig. 10.)

TABLE VII.

H. C. G. S.	Δ mm.	$H \times \Delta$
4,000	10.2	408
5,000	8.30	415
6,000	6.80	408
7,000	5.80	406
8,000	5.05	404
9,000	4.40	396
9,500	4.15	394
10,000	3.95	395
10,500	3.85	405

Aver. = 403.

The average of column three is the product of $H \times \Delta$ of the equivalent hyperbola, and this, divided by any given value of H , gives a corrected value for Δ , which may be used in determining the value of $\lambda - \lambda'$. In this way a double check is secured, first, in drawing a smooth curve through the original data, and thus getting the readings of column two; and second, in "averaging" these readings in the manner shown.

In Table VII. the average of column three is 403; the value of Δ corresponding to $H = 5,000$, is 8.06, and $\lambda - \lambda'$ is 0.144 A. U. This is given in Fig. 21.

Mercury.—Readings were obtained on the yellow, green and violet lines as follows :

TABLE VIII.

H	Yellow line. $\lambda = 5790.49$.		Green line. $\lambda = 5460.97$.		Violet line. $\lambda = 4358.56$.	
	Δ	$H\Delta$	Δ	$H\Delta$	Δ	$H\Delta$
3,000	20	600
4,000	15.	600	12.0	480
5,000	11.9	595	9.5	475
6,000	9.85	591	8.0	480
7,000	8.25	578	6.8	476	5.7	399
8,000	7.00	560	5.8	464	4.8	384
9,000	6.20	558	5.2	468	4.3	387
10,000	5.70	570	4.8	480	3.8	380
11,000	5.45	560	4.7	517	3.5	385

Aver. = 579.

Aver. = 480.

Aver. = 387.

The magnetic shift for $H = 5,000$ is, yellow line 0.128 A. U., green line, 0.155 A. U., violet line 0.120 A. U.

Lines are then drawn through zero and this value, thus giving the magnetic shift for any value of Δ . This is done on Fig. 22.

Cadmium.—Readings were taken on the red, green and blue lines as shown in Table IX., and platted on Fig. 23.

TABLE IX.

H	Red line. $\lambda = 6438.9$.		Green line. $\lambda = 5086.3$.		Blue line. $\lambda = 4800$.	
	Δ	$H \times \Delta$	Δ	$H \times \Delta$	Δ	$H \times \Delta$
3,000	15.8	475	13.9	418
4,000	18.6	745	13.	520	10.7	432
5,000	14.15	708	10.8	540	8.4	420
6,000	11.4	685	9.	540	6.85	411.
7,000	9.8	685	7.8	546	5.9	413
8,000	8.75	700	7.	560	5.3	424
9,000	8.	720	6.6	594

Aver. = 707.

Aver. = 539.

Aver. = 420.

The magnetic shift for $H = 5,000$ is red line, 0.131 A. U.; green line, 0.120 A. U.; blue line, 0.137 A. U.

The results may be summarized as follows :

TABLE X.

Substance.	Line.	Magnetic Shift for	
		$H = 5000$	$H = 10,000$
		A. U.	A. U.
Sodium. ¹	Yellow line D_1 .	0.207	0.414
Mercury.	Yellow line.	0.128	0.256
"	Green "	0.155	0.310
"	Violet "	0.120	0.240
Cadmium.	Red "	0.131	0.262
"	Green "	0.120	0.240
"	Blue "	0.137	0.274
Zinc.	Blue "	0.144	0.288

This table is shown graphically on Fig. 24.

¹ The echelon shows that the separation of the components of D_2 to be about two-thirds of that of D_1 . The value here found belongs to D_1 , the line having the greater magnetic shift.

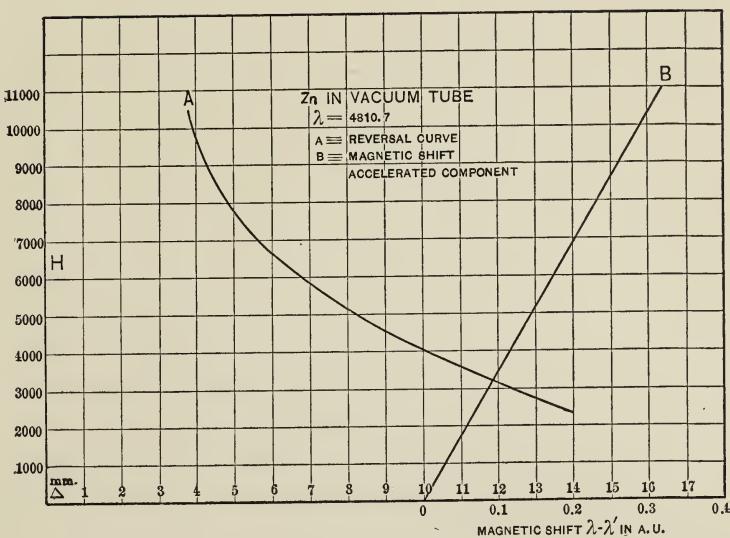


Fig. 21.

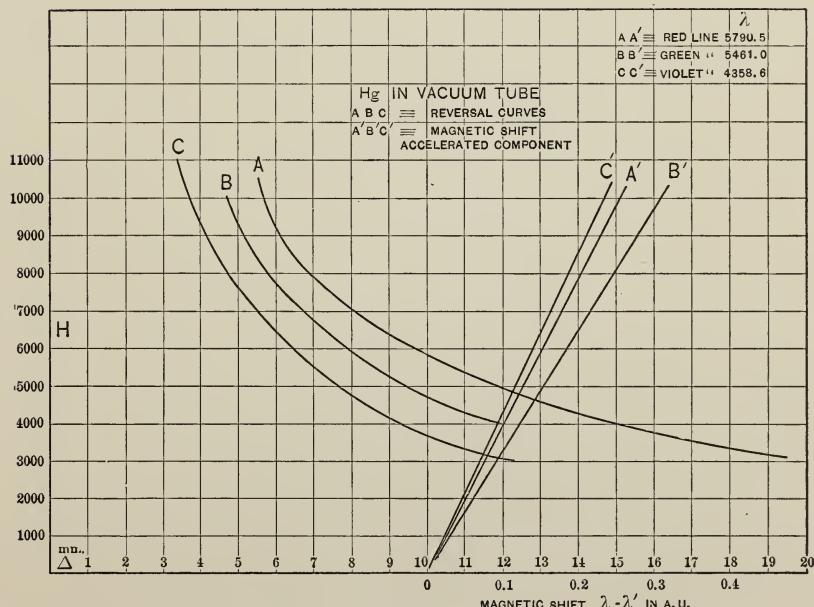


Fig. 22.

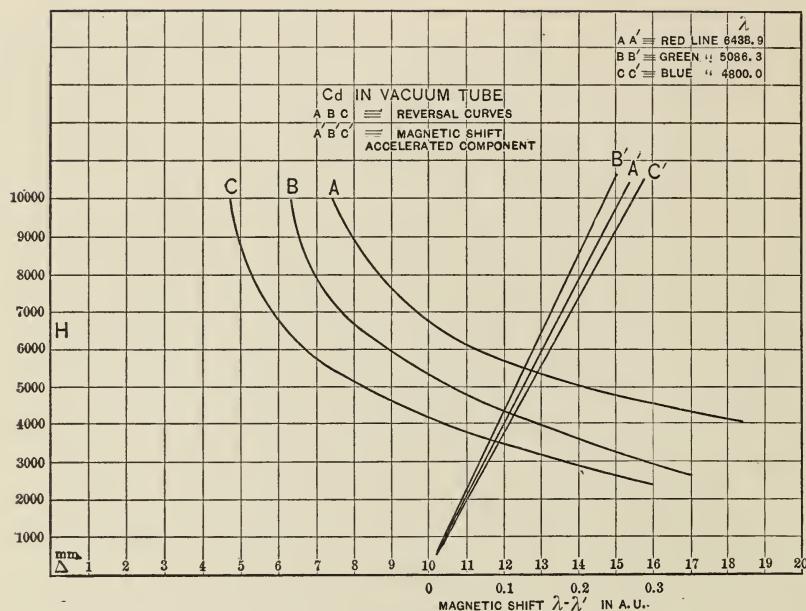


Fig. 23.

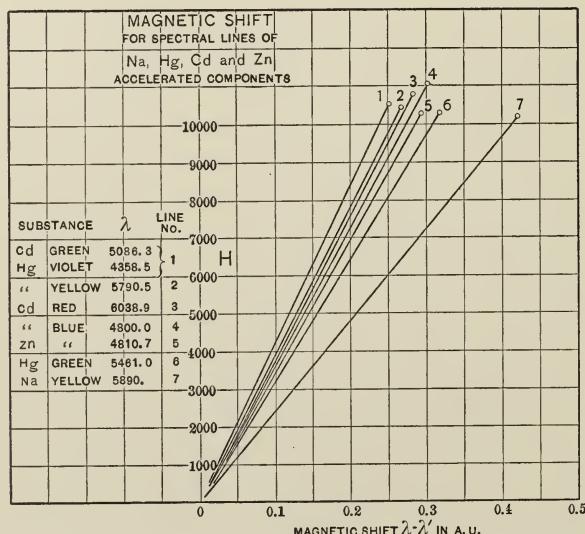


Fig. 24.

RATIO OF IONIC CHARGE TO IONIC MASS.

Equation (10) gives the relation

$$\epsilon/m = \frac{\lambda - \lambda'}{\lambda^2} \frac{4\pi v}{H} \quad (10)$$

for the case of circularly polarized light. This must be slightly modified for plane polarized light as the period of single vibration will be half that of the complete vibration of the circularly rotating ion. Equation (10) will then be written

$$\epsilon/m = \frac{\lambda - \lambda'}{\lambda^2} \frac{2\pi v}{H}. \quad (19)$$

The numerical value in electromagnetic units may now be calculated from the values of $\lambda - \lambda'$ and H .

For the lines so far examined the values are as follows: The lines may be grouped according to the value of ϵ/m ; this gives the following interesting table.

TABLE XI.

Substance.	Line.	ϵ/m	Type of line. (Michelson.)
Sodium.	Yellow.	22.45×10^5	
Mercury.	Vio et.	23.81 "	
Cadmium.	Blue.	22.41 "	Type II.
Zinc.	Blue.	23.46 "	
Mercury.	Green.	18.59 "	
Cadmium.	Green.	17.48 "	Type III.
Mercury.	Yellow.	14.35 "	
Cadmium.	Red.	11.93 "	Type I.

It is thus seen that the groupings according to Michelson's chart (see Fig. 3) and according to the value of ϵ/m are the same.

The data furnished by Preston¹ are too meager to furnish a just comparison, but it is to be said that his classification is not so well defined as is that furnished by the interferometer.

Polarization.—The method of observing the state of polarization has been outlined. In the lines examined the state of polarization

¹ Phil. Mag. (5), 45, p. 330. See part I.

was found to be normal with the exception of two cases, that seem deserving of notice.

I. *Violet Line of Mercury*.—In the readings of Table VIII. the settings were taken successively on the yellow, green and violet lines for the same value of H , by shifting the slit S in Fig. 10. The Nicol was so set as to cause the fringes to expand, thus giving the retarded component. For high values of Δ the behavior of the fringes in the case of faint lines is not readily observed, and it was not until the value $\Delta = 5.14$ mm. was reached that it was noticed that the violet line contracted, while the yellow and green lines expanded. This behavior was carefully observed for the balance of the readings and persisted throughout. This peculiar state of polarization would indicate that the component of the violet line which *agrees* in rotation with the magnetic whirl is retarded instead of accelerated, as in the other lines. Hence we must have present a species of *diamagnetic* ion in contrast to the ordinary or "magnetic" ion. The above observations were taken with the greatest of care, and yet the conclusion seemed so unusual as to call for repeated verification. All attempts, however, to secure the same observations proved unavailing so far as reversed polarization was concerned.

II. In the case of the cadmium lines the following observations were made: The *red*, *green* and *blue* lines were examined, the Nicol being set so as to cause the fringes to contract, thus giving the accelerated ray. The observations were begun with $H = 2,900$ C. G. S. units. At the second reading with $H = 6,600$ the fringes *all* expanded, and so also for $H = 7,900$. At the last reading with $H = 8,800$ *all* the fringes again contracted. The apparent conclusion to be drawn is that a reversal of the state of polarization takes place during the progress of the experiment. It is more likely, however, that in this case, since (as already stated) the glass of the vacuum tube acts as a retardation plate, some change of stress due either to variation in temperature or to the magnetic field, is responsible for the observed effect. This fact, in connection with the difference in wave-length between the violet and the green and yellow lines of mercury, may, perhaps, also explain the unusual behavior observed in this line. Both these cases illustrate the necessity of carefully watching the *instrumental factors* present.

Summary.—The study of the magnetic shift and the values of e/m lead to the following conclusions :

- I. A classification of lines according to the amount of magnetic shift is of little value.
- II. A classification of lines according to the value of e/m is significant.
- III. Such a classification groups the lines according to the type of line produced, as given by the analysis of visibility curves.
- IV. The smaller the value of e/m the less the broadening of the component lines and the simpler their structure; *vice versa* the larger the value of e/m the more the broadening, and the more complex their structure.

RECAPITULATION.

In the preceding pages the history of the subject has been traced, from the first experiments of Faraday to the present time. The bearing of the Maxwell-Lorentz theory upon the progress of Zeeman's experiments is noted, and the reacting influence of experiment upon theory is shown, so that in the papers of Larmor, Lorentz and others the mathematical exposition of the subject has kept pace with the experimental observations. In the experimental development of the problem two well-marked methods have been presented and compared. It has been found that the spectro-photographic method is (1) limited in range by reason of the small resolving power of the ruled grating when so minute changes are to be measured; and (2) is limited in accuracy by reason of the wide margin of error in the settings of the micrometer, especially when nebulous lines are to be measured.

On the other hand, the interferometer method has been seen to possess a resolving power greatly in excess of the photographic method, and hence is applicable to low values of H as well as to high values.

The results accomplished so far consist in

- I. By the spectro-photographic method.
 - (1) A classification of lines according to the type of line produced by the magnetic field.

(2) The measurement of the distance between outside components of magnetic triplets and a determination of the value of the ratio e/m .

The number of lines so far examined is not large, nor do the values of the magnetic shift obtained by different observers seem to agree very well; but enough has been done to suggest a classification of lines parallel to that obtained by Kayser and Runge.

II. With the interferometer Professor Michelson has presented,

(1) Three well-marked types of lines, with a possible fourth type.

(2) He has concluded that the magnetic shift is to be regarded as approximately independent of substance or color.

The chief value to be attached to his work is contained in (1), and it would seem to us better to leave (2) unformulated rather than to state it as an approximate law.

The experiments described in this paper have sought to determine,

(1) The relation, at different temperatures, of the magnetic shift to strength of field.

(2) To present a method of measuring the magnetic shift that shall be as free as possible from objections involved in existing methods.

The method adopted is similar to existing interferometer methods, but is believed to be especially adapted to the present problem.

(3) To measure the magnetic shift and determine the ratio e/m .

The values obtained for magnetic shift show that any classification based upon this alone is of relatively small value; but that a classification based upon the value of e/m is significant.

This classification is seen to give groups of lines identical with the groups presented by Michelson's three types of lines.

The complexity of structure is also seen to depend upon the value of this ratio e/m .

The preceding experiments were carried on in connection with the Fellowship in Physics from 1897 to 1899 at the University of Wisconsin, under the direction of Professor Benjamin W. Snow; whom, in conclusion, I desire to thank for his helpful and continued encouragement throughout the progress of this investigation.

APPENDIX I.

Extract from Life of Faraday by Dr. Bence Jones, Vol. II., p. 44.
“ 1862 was the last year of experimental research ; Steinheil’s apparatus for producing the spectrum of different substances gave a new method, by which the action of magnetic poles upon light could be tried. In January he made himself familiar with the apparatus, and then he tried the action of the great magnet on the spectrum of NaCl, BaCl, StCl and LiCl,

“ On March 12 he writes :

“ ‘Apparatus as on last day (January) but only ten pairs of voltaic battery for the electromagnet.

“ ‘The colorless gas flame ascended between the poles of the magnet, and the salts of sodium, lithium, etc., were used to give color. A Nicol’s polarizer was placed just before the intense magnetic field, and an analyzer at the other extreme of the apparatus. Then the electromagnet was made, and unmade, but not the slightest trace of effect on or change in the lines of the spectrum was observed in any position of polarizer or analyzer.

“ ‘The other pierced poles were adjusted at the magnet, the colored flame established between them, and only that ray taken up by the optic apparatus which came to it along the axis of the poles, *i. e.*, in the magnetic axis or line of magnetic force.

“ ‘Then the electromagnet was excited and rendered neutral, but not the slightest effect on the polarized or unpolarized ray was observed.’

“ This was the last experimental research that Faraday made.”

II.

Extract from M. Ch. Fievez (Astronome à l’Observatoire Royale de Bruxelles), *Bulletins de l’Academie Royale de Belgique*, 3d serie, tome ix., p. 381 (1885).

“ L’installation spectroscopique de l’Observatoire, disposant d’un appareil dispersif de très grande puissance et d’un électro-aimant Faraday, construction Ruhmkorff, pouvant être actif par un courant de 50 ampères d’intensité, a permis d’aborder ce problème.

“ La flamme oxyhydrique d'un petit chalumeau etait dirigee horizontalement sur un charbon *sode* place entre les armatures coniques de l'electro-aimant, distantes l'une de l'autre de 10 millimetres. Une image de la flamme etait projetee sur la fente du spectroscope par un objectif double. La quantite d'oxygene introduite dans cette flamme permettait de regler la temperature de facon a donner aux raies spectrales D_1 et D_2 l'apparence voulue.

“ Dans ces conditions, les raies sodiques D_1 et D_2 etant d'abord peu larges et non renversees avant le passage du courant d'aimantation, deviennent immediatement *plus brillantes, plus longues et plus larges* aussitot que l'electro-aimant est mis en activite.

“ Si les raies brillantes D_1 et D_2 sont deja elargies, l'electro-aimant etant inactif, elles deviennent plus larges encore et *se renversent* (c'est-a-dire qu' une raie noire paraît au milieu de la raie brillante elargie) pendant le passage du courant d'aimantation.

“ Si les raies sont deja elargies et renversees, l'elargissement de la raie brillante et de la raie noire devient beaucoup plus considerable.

“ Ces phenomenes, qui disparaissent instantanement lors de l'interruption du courant, peuvent etre observes, mais avec moins d'intensite, sur la raie rouge du potassium, du lithium, sur la raie verte du thallium, etc., lorsqu' une minime quantite de ces metaux ou d'un de leurs sels est placee sur le support de charbon.

“ Enfin, les armatures coniques de l'electro-aimant etant remplacées par les armatures meplates, de maniere que toute la longueur de la flamme sodique soit comprise entre ces armatures, les raies D_1 et D_2 , prealablement renversees et elargies, presentent un *double renversement* (c'est-a-dire l'apparition d'une raie brillante au milieu de la raie noire elargie), lorsque l'electro-aimant est en activite.”

III.

Extract from article in Astro. Phys. Jr., February, 1899, by Perot and Fabry.

“ * * * when the light is complex it is easy to obtain a precise measure of the ratio of the wave-length of the radiations which constitute it; let there be two radiations of nearly equal wave-length λ and $\lambda - \epsilon$. The distances between the silvered surfaces is in-

creased until the discordance between the two systems of rings is complete. Then if e is the distance between the surfaces (which is given with sufficient accuracy by the micrometer) we have

$$\frac{\varepsilon}{\lambda} = \frac{\lambda}{4\varepsilon}$$

"* * * This method is also readily adapted to the study of the change of wave-length of a given line, on condition that the radiation be sufficiently monochromatic; in such a case a comparison can be made of two sources emitting, for instance, in the one case the altered radiation and in the other the normal radiation, attention being directed to the change in the appearance of the rings produced by the two sources successively."

* * * * * *

NOTE.—With the interferometer the ray travels the distance between the plates twice and hence $\Delta = 2e$. Therefore the equation for the interferometer becomes

$$\frac{\varepsilon}{\lambda} = \frac{\lambda}{2\Delta} \text{ or } \varepsilon = \frac{\lambda^2}{2\Delta}.$$

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